Exact Riemann Solution for the Euler Equations with Nonconvex and Nonsmooth Equation of State

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Abstract

This thesis is concerned with the solution of the Riemann problem for the Euler equations with nonconvex and nonsmooth equation of state. The main goal is to develop an exact Riemann solution by means of wave curve composition, which accounts for physically relevant effects such as composite waves or split waves across phase boundaries.

A frequently considered experimental configuration is a shock tube, where two states of a fluid are initially separated by a diaphragm. Instantaneously removing the diaphragm leads to a development of various physical states embedded in moving separated regions inside the tube, where the fluid is compressed or rarefied in a specific manner. In contrast to the standard setting, where only shock waves and rarefaction waves exist, so-called BZT-fluids possess a region of negative nonlinearity in phase space. This leads to an additional wave type, composed of shock and rarefaction parts or to expansion shocks still satisfying the second law of thermodynamics. When we additionally take into account phase transitions, i.e., in our context transitions from vapor to liquid or vice versa, further non-classical effects such as wave splitting at the phase boundary may occur.

In general the solution of a shock tube problem in space–time plane can be considered as a projection of wave curves in phase space. In short, each wave curve collects states which can be reached in a certain sense from its origin. If initial data is given, finding a solution to the shock tube problem means to determine the wave curves and the intermediate states, such that the wave curves form a way from one initial state to the other. Once the wave curves are constructed, the solution of the shock tube problem is known for all times due to the scale-invariance of the system. Here most of the mentioned aspects concerning solutions of a shock tube problem arise one–to–one for general hyperbolic systems of conservation laws.

In summary, this thesis contains: (I) a *construction principle* in order to analytically construct the (Liu) entropy solution for the shock tube problem by composing waves, including the possibility of wave curves crossing phase boundaries and regions with negative nonlinearity; (II) an *existence and uniqueness proof* for the additional composite wave curve type occurring in the construction principle; (III) *algorithms* embedded in a library containing the exact Riemann solver to compute the solution for a shock tube problem numerically; (IV) *comparisons of solutions* of several numerical schemes for various configurations with the respective exact solution and the answer of the question, whether the schemes are capable to catch all new wave phenomena such as wave splitting.